A paper entitled, "An Inquiry as to the Coefficient of Labouring Force in Overshot Water-Wheels, whose diameter is equal to, or exceeds the total descent due to the fall; and of Water-Wheels moving in Circular Channels," was read by Robert Mallet, Esq., Mem. Ins. C. E., M. R. I. A.

This paper is partly mathematical and partly experimental. The investigation which it describes, the results of which are given in ten tables, had in view principally to obtain definite experimental answers to the following questions:

1st. With a given height of fall and head of water, or in other words, with a given descent and depth of water in the pentrough, will any diameter of wheel *greater* than that equal to the fall give an increase of labouring force (i. e. a better effect than the latter), or will a loss of labouring force result from such increase of diameter?

2nd. When the head of water is necessarily variable, under what conditions will an advantage be obtained by the use of the larger wheel, and what will be the maximum advantage?

3rd. Is any increase of labouring force obtained by causing the loaded arc of an overshot wheel to revolve in a closely fitting circular race or conduit, and if so, what is the amount of advantage, and what the conditions of maximum effect?

The author briefly reviews the history of our knowledge of this branch of hydrodynamics, the experimental researches of Da Borda, Smeaton, &c., and the more recent improvements in the theory of water-wheels, due to the analytic investigations of German and French engineers, and the admirably conducted experiments of Poncelet, Morin, and the Franklin Institute.

Smeaton, in his paper on water-wheels, read to the Royal Society in May, 1759, and Dr. Robison of Edinburgh, in his treatise on water-power, lay down as a fixed principle, that no advantage can be obtained by making the diameter of an over-

shot wheel greater than that of the total descent, minus so much as is necessary to give the water a proper velocity on reaching the wheel. The author, however, contends that the reasoning by which the latter writer upholds this is inconclusive,—that there are some circumstances which he points out necessarily in favour of the larger wheel, and that conditions may occur in practice in which it is desirable to use the larger wheel, even at some sacrifice of power; and that hence it is of importance to ascertain its value in use, as compared with Smeaton's size for maximum effect.

The author states the general proposition, "that the labouring force (*travail* of French authors, or mechanical power of Smeaton) of any machine, transferring the motive power of water, is equal to that of the whole moving power employed, minus one-half of the *vis viva* lost by the water on entering the machine, and minus one-half of the *vis viva* due to the velocity of the water on quitting it." He then obtains general equations expressing the relations between the fall, the velocity, the weight of fluid, the power, &c. in overshot water-wheels, at whatever point the water may first reach the wheel, and whether the latter move naked, or in a circular channel or course. From these he deduces, that—

1st. If the portion of the total descent passed through by the water, before reaching the wheel, be given, the velocity of the circumference should be one-half that due to this height.

2nd. If the velocity of the circumference be given, the water must descend through such a fraction of the whole fall, before reaching the wheel, as will generate this velocity.

3rd. The maximum of labouring force is greater as the velocity of the wheel is less, and its limit theoretically approaches that due to the whole fall; general equations are then given, expressing the amount of labouring force in all the conditions considered by the author, and their maxima.

One of the principal advantages of using an overshot wheel *greater* in diameter than the height of the fall, is the

capability thus given of making any additional head of water occurring at intervals, by freshes or any other cause, available, by letting the water on the wheel at higher and higher levels.

The first course of experiments is devoted to the determination of the comparative value of two water-wheels, the one whose diameter is equal to the whole fall; the other to the head and fall, or to the total descent. By the head, the author always means the *efficient head*, or that due to the *actual velocity* of efflux at the sluice or shuttle, as determined by Smeaton's experimental method,—this was equal to six inches in all cases.

The apparatus employed in these researches consisted of two accurately made models of these wheels, with curved buckets, made of tin plate, the arms, &c. of brass, and the axes of cast iron, working on brass. Special contrivances were adopted to measure the weight of water passed through each wheel in each experiment, which was in every case 1000 lbs. avoirdupoise; and others, to preserve the head of water quite constant,—to determine the number of revolutions made per minute, and thence the speed of the wheels. One wheel was 25.5 inches diameter, the other 33 inches diameter. The value of the labouring force was determined directly by the elevation of known weights to a recorded height by a silken cord over a pulley; the altitude was read off, on a fixed rule placed vertically against a lofty chimney. The relative power of the wheels was determined by the speed of rotation of a regulating fly or vane.

All the principal results given in the ten tables are the average of *five good experiments.* From the accurate workmanship and large size of these models, the peculiar contrivances for ensuring accuracy of observation, and the care taken in the experiments, the author reposes considerable confidence in his results as practical data.

The velocity, in reference to maximum effect, is determined,

and found to be lower than that deduced by Smeaton from his experiments, which the author presumes arises from the better construction of apparatus, and better form of bucket used in the present case.

The author then ascertains, by another train of experiments on both wheels, the value of the circular conduit or race, and finds, in round numbers, that there is an economy of labouring force, amounting to from eight to eleven per cent. of the power of the fall, obtained by its use. This conduit acts by retaining the water in the buckets at the lower portion of the loaded arc. The velocity of a water-wheel working thus, he finds may vary through a greater range without a material loss of power than when working naked, and that a steady motion is also continued to a much lower velocity.

The author arrives at the following practical conclusions:

1st. When the depth of water in the reservoir is invariable, the diameter of the water-wheel should never be greater than the entire height of the fall, less so much of it as may be requisite to give the water a proper velocity on entering the buckets.

2nd. When the depth of water in the reservoir varies considerably and unavoidably in depth, an advantage *may* be obtained by applying a larger wheel dependent upon *the extent of fluctuation* and *the ratio in time* that the water is at its highest and lowest levels during a given prolonged period; if this be a ratio of equality in time there will be no advantage, and hence in practice the cases will be rare where any advantage will be obtained.

3rd. If the level of the water in the reservoir never fall below the mean depth of the reservoir, when at the highest and lowest, and the average depth be between an eighth and a tenth of the height of the fall, then the *average labouring force* of the large wheel will be greater than that of the small one, and it will of course increase this advantage at periods of increased depth of reservoir.

Hence the author affirms that Dr. Robison's conclusions must henceforth receive a limitation.

Having shown that a positive advantage is obtained by the use of the circular conduit, amounting to about eleven per cent. of the total power, and that this value increases with an increase in the velocity of the wheel up to six feet per second, or more in large wheels, the author contends, that it is practicable to increase the efficiency of the best overshot wheels as now usually made, at least ten per cent. by this application. The only objections ever urged against the conduit were of a merely practical character, and the author shows that improved workmanship, and the modern use of cast iron, of which the conduit may be constructed, and provided with adjustments, render these no longer tenable.

Drawings of the apparatus used in these researches, and the tabulated results, were exhibited to the Academy.

Professor Lloyd read a paper "on the Phenomena of Thin Plates in Polarized Light."

The author stated, that his attention had been drawn to this subject by a letter which he had received from Sir David Brewster, describing a large class of hitherto unobserved phenomena. Sir David Brewster having expressed his desire, in this letter, to know whether the wave-theory could furnish an explanation of the facts which he had observed, Professor Lloyd was thus led to undertake the investigation which formed the subject of the present communication.*

Mr. Airy had long since inferred, from a consideration of the form of Fresnel's expression for the intensity of reflected light, that when light, polarized perpendicularly to the plane

* The present paper was read in the Mathematical Section of the British Association, last year; and a summary of the results was published in the *Athenæum*, of August, 1841. The author deferred submitting it to the Academy, in the hope of being able to add an experimental confirmation of some of the conclusions not noticed by Sir D. Brewster. He has, however, been compelled, by the pressure of other duties, to postpone still further this branch of the investigation.